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THE POSTULATES OF DEDUCTIVE LOGIC

ONE of the greatest dangers to cogency in thinking is the intrusion of irrelevant ideas; and one of the great advantages of mathematical notation has been that it helps to exclude irrelevancies. This advantage is much increased, now that mathematical scholars are generally aware that their symbols mean no more than the postulates and definitions make them mean. For the common-sense associations which formerly did so much to introduce unwarranted assumptions into the reasoning, especially in geometry and mechanics, are now effectually guarded against, although they still serve their old function of suggesting possible combinations.

There are two means by which the meaning of symbols can be fixed. One is definition in terms of symbols whose meaning is already fixed. The other is the exhibition of a set of standard formulæ in which the symbols in question occur, and which constitute the authority for their use on all occasions. Both methods are used in our dictionaries in fixing the meaning of the words of the various languages. It is obvious that in the case of words the latter method is the fundamental one. It is common usage that gives words their meanings, and the definitions have to be framed accordingly.

These same methods are followed in mathematics; but the endeavor is made to give the definitions and standard formulæ such exactness and clarity that no error in their application can occur except by reason of inattention or forgetfulness; and, furthermore, the two methods are treated as mutually exclusive alternatives. Definition is used wherever possible; and only where definition can not be used is resort made to a set of standard formulæ—the postulates.

But the usage of words in ordinary language is not simply an employment of words in various combinations with each other. It is the employment of them in appropriate situations. It is from the external connection of words with things and feelings that, in the last resort, all the meaning of words is derived. Now in mathematical procedure all such external connections are *as far as possible* avoided. It is the standard usage of the symbols in connection with each other that is expected to guide all further usage.

The various branches of mathematics may be viewed as a series of deductive systems, presupposing one another in a certain definite order, and with each later system characterized by a new set of postulates in which new "indefinables" occur. Thus geometry, while presupposing arithmetic, may be viewed as requiring for its establishment ideas and assumptions that are foreign to arithmetic. This conception of geometry was, in fact, universal until recently, and it is still very frequently adopted. It is now, however, generally recognized that this conception is not necessary—that geometry may be so universalized that it contains no radically new ideas or principles at all, and thus takes its place as a part of pure mathematics, resting upon no other basis than the postulates of deductive logic.

Now in the organization of deductive logic itself as a deductive science, it is such sciences as geometry that have served as the pattern. Perhaps, therefore, it may help us to a better understanding of the postulates of deductive logic, to consider briefly what has recently been made clear with regard to the logical status of the postulates of geometry.

That these postulates are not asserted propositions has long been recognized. The old controversy over the "parallel-axiom" of Euclidean geometry is responsible for that. The "axiom" might be assumed, or an alternative might be assumed, and geometry went on its way quite merrily in either case. It therefore came to rank as an assumption with whose truth or falsity geometry was not concerned; but it still remained open to question whether it was indeed true or false. For a time, indeed, the case of this "axiom" was regarded as exceptional. But it was finally seen that this was not so—that any of the "axioms" might be altered or replaced, and a geometry of some sort still remain. A further step was involved in the recognition of the possibility of diverse interpretations of the assumptions, some of which might be true and others false. Thus the "parallel-axiom," whether it be true or false in its common application to physical space, is certainly true in its analogous application to the "space" whose "points" are ordered sets of real numbers. The science, meanwhile, remained perfectly indifferent to these different interpretations. It concerned itself only with deducing the consequences of the postulates in their symbolic form. The postulates were no longer capable of truth or falsehood. They were conventions subject to the arbitrary choice of the mathematicians.

This view, however, was found to be short-sighted. In the first place, such a postulate as the "parallel-axiom," even though it be expressed in peculiar symbols that are not found in ordinary dictionaries, is not thereby wholly cut off from those external connections which, as has been said, give words their meanings. A set of geometrical postulates need contain (or presuppose) only two indefinables that are peculiar to the subject: say, *point* and *between*. All the other indefinables are such as occur in all intelligent discourse. These others, in the formal statement of the postulates, may be expressed in common words, or new symbols may be invented for them; but if new symbols are used they must first be explained in ordinary language. The use of new symbols to express these ideas could only have the advantages of brevity and (perhaps) of unambiguity. Thus, even anterior to any particular interpretation, the postulates are not wholly unintelligible. Suppose, for example, we read: "If *p, q,* and *r* be each any ϛ, and if *p* ∠ the collection *q* and *r,* then *q* does not ∠ the collection *p* and *r*." Now this may be the first time we have ever seen the symbols ϛ and ∠, and we may not have the faintest notion how to pronounce them. But we know the meaning of all the words and punctuation marks that occur; we know that ϛ is a class-name and that ∠ stands for some relation; and we know that *p,* like *q* and *r,* is intended to denote the same ϛ throughout. If this be one of a set of postulates, we observe further that these same symbols appear in the other postulates, and that the only connection in which ϛ's are mentioned is either their mere existence or their occurring (or not occurring) in the relation ∠.

Hence, instead of the postulates being either (1) propositions asserted to be true; or (2) assumptions, which may be true or false, but which are made in disregard of their actual truth or falsity; or (3) mere formulæ, which are meaningless till some particular interpretation is given them; they are more properly to be regarded (4) as the perfectly intelligible description of a certain class of relations, or, what amounts to the same thing, of the entities which enter into this class of relations. For example, the above postulate might be expressed: "Let us consider the class of relations which a single term may bear to a pair of terms, and which are asymmetrical as respects the single term and either member of the pair." The remaining postulates would give further particulars of the description; or, by providing for the existence of terms between which such relations obtain, they would by implication assert that all the various parts of the description are consistent with one another. In present-day practise, however, the consistency of the description is always proved by pointing to some actual exemplification; and the example is always, in the last resort, some combination of numbers.

But the complete description of a class of relations is its definition. Hence, in the case of geometry, the "indefinables" are not absolutely such. They are simply assumed as indefinable for the science. And the set of postulates, by means of which the employment of the "indefinables" is fixed, reduces upon examination to an instance of the other mode of fixing the employment of symbols.

How is it, then, with the postulates of the fundamental branch, or rather trunk, of mathematics—deductive logic? Their difference from geometrical postulates appears in the fact that if uninterpreted symbols are substituted for the logical terms, nothing but rows of symbols remains. These can not, then, be translated into a general description of something or other, for there is nothing to take hold of upon which a translation might be founded.

Can they then be treated as a set of symbolic formulæ awaiting interpretation? Let us see.

Imagine a sheet of paper covered with a variety of marks which are repeated in different combinations in different lines. Imagine, further, a set of directions setting forth the conditions under which anyone may, if he please, write similar lines of marks on an indefinite number of other sheets of paper. These directions must be extraordinarily explicit, no possible room for misunderstanding being left open; and they must, of course, be expressed in intelligible language. By this I do not mean that they will contain nothing but words; for they must contain the marks the employment of which they direct. The first direction will be that nothing shall be set down except as shall be thereafter distinctly permitted; all the remaining directions will express positive conditions.

Under these circumstances you might proceed to cover a considerable number of blank sheets with these (to you) meaningless marks. You might develop much cleverness in seeing possible combinations; and you might thus devise new directions leading by short-cuts to results which the original directions invariably authorized. The devising of these short-cuts would perhaps take up a very large part of your efforts; and you might fill up many note-books with your record of them, and your proofs that they did indeed involve no transgression of the original directions. These proofs you would conduct as your mother-wit enabled you; and if you had ever studied logic you would doubtless check them up by such logical rules as you were acquainted with.

You might do all this without having the faintest idea what the marks meant or whether they meant anything at all. And then it might accidentally occur to you—I do not know how you could set about systematically to make such a discovery—or some one who already knew might call your attention to the fact, that those original rows of marks might be given a meaning: that they might be understood to indicate certain principles of logic; and that the directions laid down for making new marks would then be equivalent to a second set of logical principles, according to which inferences might be drawn from the first set. At the same time you would bethink yourself that, for all you knew, the marks might be susceptible of an infinite number of other interpretations, and that the directions for making new marks might have to be variously interpreted accordingly.

This, so far as I know, would be a perfectly feasible enterprise. The directions for making new marks would have to cover in some way: (1) inference of the form, "p and 'p implies q' permit us to write q"; (2) substitution of variables for other variables whose range includes theirs (as, for example, "p is false" for p); (3) the use of parentheses or their equivalent, and the occasional dropping of parentheses; (4) the arbitrary framing of definitions and the substitution of either side of a definition for the other.

Such an enterprise would not only be feasible, but it might have a certain utility. For there would be a minimum of distraction by irrelevant associations, and hence a minimum danger of being led into unwarranted assumptions. The pages upon pages of rows of marks that were written would not have to be changed by an iota when the interpretation was made. It must be confessed that it would probably take a phenomenal intellect to get very far in the development, when all help from common experience was to this extent eliminated; and that such an intellect might very likely be better occupied.

But whatever utility such a treatment might have, we must not be blinded to the fact that the whole procedure would be *absolutely nonsensical* until the interpretation of the original marks was given; and that this interpretation is, from the standpoint of the manipulator of the marks, an *absolute accident*—as accidental as the interpretation of a well-fought game of chess as an argument for the immortality of the soul. And meanwhile the very manipulation of the marks themselves rests, of course, on the use of common speech in the directions; so that the natural imperfection of language, which has been pitchforked out of the rows of marks, flies back through this window.

It has been said that the enterprise of working out a mathematical logic without knowing what it meant, would require a far greater intellect than is likely ever to be applied to it. This circumstance, however, need not detract from a certain value which the very conception of such an enterprise may have for us. We may use this conception in the critical examination of the postulates of any proposed system of mathematical logic, asking ourselves whether they would equally suffice if abstraction were made from their significance; that is to say, whether certain of the postulates can be regarded as a set of meaningless marks, and the remaining postulates as a full set of directions for the writing of additional marks.

Thus the postulates of deductive logic stand upon a very different footing from the postulates of geometry. To geometry the interpretation of its primitive symbols is a matter of indifference—except that the possibility of at least one interpretation is necessary in order to ensure the mutual compatibility of the postulates. For deductive logic the interpretation of the symbols is logically prior to all else, for without that all else is nonsense.

Accordingly, it is impossible for deductive logic to cut itself entirely loose from the external connections of common language and its consequent unclearness. It should, indeed, be the aim of the logician to reduce this unclearness to the practicable minimum by the methods of the dictionary, troubling himself not at all by reason of the logical circles into which he is thus led. Imperfect as the explanation may necessarily be, it is an essential propædeutic to the system of logic—not, as some writers would have it, a mere concession to practical expediency. At the same time it is not to be forgotten that the postulates themselves, when once some vague meaning has been given to their terms, are a most potent means to the further clarification of their meaning.

I wish now to discuss two of the postulates, which, in certain recent systems of deductive logic, stand out prominently by virtue of the fact that they are expressed in words and not in quasi-mathematical symbols; and I wish particularly to examine the reasons that are given for the use of words. The two postulates are the *principle of deduction* and the *principle of substitution.*

M. Couturat, in the syllabus of logic which he has contributed to the "Encyclopedia of the Philosophical Sciences," states the former principle as follows: "If A implies B, and if A is true, then B is true (and we can affirm this independently)." And he adds: "We should be tempted to express this principle by the formula:

$$(A < B)(A = 1) < (B = 1),$$

but this formula is again an implication from which we can not extract the thesis $(B = 1)$ in order to affirm it separately except in virtue of this principle itself. And here we have a striking proof of the necessary limitation of symbolism when it is a question of formulating principles" (Vol. I, p. 143). To this I would reply:

(1) The principle of deduction, as M. Couturat has himself stated it in words, is itself an implication, from which the thesis "B is true (and we can affirm this independently)" remains to be "extracted"; and this, according to M. Couturat's own words, can only be done "in virtue of this principle itself." It may be added that either in the case of the symbolic formula or in that of the verbal statement, it would be well to leave the thesis unextracted; for, standing by itself, it is in general untrue.

(2) The symbolic formula, which M. Couturat suggests only to withdraw it, is in fact precisely equivalent to his verbal statement of the principle of deduction. The symbol "$<$" means "implies," or "if . . . then"; the symbolic expression "$=1$" means "is true"; and the placing of the two parentheses side by side means "and." Accordingly, the whole formula reads: "If A implies B, and A is true, then B is true." The only addition in M. Couturat's verbal statement of the principle is the phrase, "and we can affirm this independently." But even this is no real addition (as M. Couturat covertly recognizes by placing it in parentheses[1]); for the stating of propositions "independently" is our common mode of stating them as true.

In "Principia Mathematica" by Messrs. Whitehead and Russell, the principle of deduction is first stated with respect to what are there called "elementary propositions." They write: "*1.1 Anything implied by a true elementary proposition is true. Pp." (This last is the symbol for "primitive proposition.") And they comment as follows: "The above principle will be extended in *9 to propositions which are not elementary. It is not the same as '*if* p is true, then *if* p implies q, q is true.' This is a true proposition, but it holds equally when p is not true and when p does not imply q. It does not, like the principle we are concerned with, enable us to assert q simply, without any hypothesis. We can not express the principle symbolically, partly because any symbolism in which p is variable only gives the *hypothesis* that p is true, not the *fact* that it is true" (p. 99).

Now (1) the principle of deduction, as these writers enunciate it, is, in fact, precisely the same as the proposition from which they attempt to distinguish it. In the first place, the transformation from the one form to the other is simple and obvious. Let us take an analogous case: "Any son of a good man is good." This means the same as: "If y is a son of a good man, y is good." And this in turn means the same as: "If x is a good man, then if y is a son of x, y is good."

In the second place, the alleged difference between the two statements is illusory. When our authors say of the second that "it holds good equally when p is not true and when p does not imply q," this seems to mark a difference, especially since p and q do not occur in the first. But, as a matter of fact, no matter what p and q may mean, and no matter what relation is fancied between them, the first statement holds. It holds whether or not pigs have wings, and whether or not the Bishop of Barchester is henpecked. In other words, being true it is true—exactly like the second.

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[1] I have not seen the original French of M. Couturat's article, and the translator of this volume is not altogether trustworthy. But I have before me the statement of the principle of deduction in his "Les Principes des Mathématiques," and this may be literally rendered as follows: "If a true implication $p \supset q$ is given, and if the hypothesis p is true, then the thesis q is true also, so that we may affirm it independently." Here the "so that" (*de sorte que*), instead of "and," more explicitly recognizes that the concluding phrase makes no real addition to the principle.

In the third place, the principle of deduction can in the one form, no more and no less than in the other form, "enable us to assert q simply without any hypothesis." Neither form will enable us to assert q by being used *as a general formula in which to substitute particular values.* Consider the propositions, "This is red," and "That is blue"; and suppose the first is true and implies the second. Then let the attempt be made to substitute these values in the principle of deduction. In the first form of the principle, substitution can not be made directly, because in it the variables are hidden under class-names. It must be put into some such form as the second, before substitution is possible. And then, when the substitution is made, we have merely: "If this is red, then, if 'This is red' implies 'That is blue,' that is blue." But in this proposition the truth of "This is red," and of the implication of "That is blue" by "This is red," is no longer asserted: it is only considered as a possibility. Hence, also, the truth of "That is blue" is not asserted.

On the other hand, of course, either form of the principle will "enable us to assert q simply," when it is used *as one premise of a deductive argument,* conducted according to the principles which govern such reasoning. We may, for example, argue: "Anything implied by a true elementary proposition is true. 'This is red' is a true elementary proposition. Therefore anything implied by 'This is red' is true. But 'That is blue' is implied by 'This is red.' Therefore 'That is blue' is true." But this is far from our authors' intention.

(2) The statement that the principle of deduction can not be stated in symbols is, accordingly, false. In the symbolism employed by our authors, it reads:

$$\vdash :. \, p \supset : p \supset q \, . \supset . \, q$$

which is, indeed, the direct translation of the second form of the principle considered above. (The first symbol is the sign of "assertion"; $\supset$ means "implies"; and the dots are used instead of parentheses.) Their remark, that "any symbolism in which p is variable only gives the *hypothesis* that p is true, not the *fact* that it is true," is simply beside the point. For in the principle of deduction we do not expect to find asserted the truth of any other proposition than the principle itself.

I conclude, therefore, that the reasons which M. Couturat and the authors of "Principia Mathematica" have given for the use of words in the statement of this logical postulate are unsound. The fact, nevertheless, remains that in building up their systems they have found themselves compelled to state it in this way—even though they had stated it in symbols elsewhere—and it is this fact that has led these acute thinkers into giving bad reasons for it.

The real reason for the fact, I believe, lies in the considerations which were presented in the first part of this paper. When once the fundamental premises of a system of logic have been set forth in symbolic form, the whole further construction of the system is carried through exactly as if the symbols had no meaning at all, or as if their logical meaning were only one of an infinite number of possible meanings. There must, therefore, be given a set of rules for the manipulation of the symbols; and these must be stated in intelligible language. For the authors whom we have been studying, the principle of deduction is one such rule.[2] When, however, the meaning of the symbols is recalled, the process of manipulating them is seen to be a process of deductive inference, and the rules of the process are seen to be principles of inference—just such principles as the whole system of logic is made up of.

So long, then, as the meaning of the symbols is borne in mind, is there any point in the separate statement of certain of the principles in words? I think not.

In speaking of the rules for the manipulation of symbols, attention was drawn to the obvious fact that in the use of these rules a man would have to reason. This necessity is not the least bit lessened when the rules are interpreted as principles of inference. No amount of knowledge of logic can enable us to dispense with thinking, though it may enable us to economize it. To be sure, this thinking, so long as it is correct, is such thinking as is in accordance with the principles of inference. But in order to justify it by those principles, *more* thinking must be done; and so on *ad indefinitum.* Thus the setting-forth of a set of principles of inference, by which deductions shall be drawn from another set of principles of inference, strikes me as a vain complication.

The authors of "Principia Mathematica," in the summary of *2 and under *2.38, distinguish carefully between the use of a proposition as a *premise* and its use as a general formula in which a particular substitution is to be made. (In this latter use, they speak of the proposition as a "principle of deduction" or "rule of inference," which seems to me confusing, if not confused.) But in their system the verbally expressed postulate, which, following M. Couturat, we have called the "principle of deduction," is not used in either of these two ways. How is it used? It is a thousand times referred to, but it is not used at all.[3] It does, however, correctly (though only partially) describe a process of inference that must continually be performed in building up the system.

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[2] It should be noted that in "Principia Mathematica" the comprehensive statement of the principle is not given before * 9.12: "What is implied by a true premise is true. Pp."

Only a few words need be added on the subject of substitution. In "Les Principes des Mathématiques," M. Couturat wrote: "That is not all: to this principle [of deduction] it is necessary to add the *principle of substitution:* 'In a general formula, a general or indeterminate term may be replaced by a particular or individual term.' That is evident, since a general formula has no value at all or even sense except in so far as it can be applied to particular terms. This principle, like the last, can not be translated by symbols, just because it is at the basis of the use of symbols; and, indeed, one could only express it, including the notion of 'particular term,' by means of general symbols [Strange that he did not see that this is equally true of the verbal expression!]; but to apply it to terms that are really particular, it would be necessary to substitute these for the general terms which represented them in the formula, which can only be done by virtue of the principle itself."[4]

On the other hand, "Principia Mathematica" gives no directions that specify when and how the process of substitution is possible. It merely declares: "In such cases [*i. e.*, where it is noticed that certain propositions are instances of general rules already given], these rules are not premises, since they assert any instance of themselves, not something other than their instances." This is almost too witty to be correct. When, for example, in *2.1, not-p (*i. e.*, the denial of the elementary proposition p) is substituted for p in the formula, "'p or p' implies p," making "'Not-p or not-p' implies not-p," it must not be supposed that the latter is *an instance* of the former. The truth, of course, simply is that every instance of the latter is an instance of the former; and that is true because every instance of not-*p* is an instance of *p* (as the postulate * 1.7 asserts). The process of substituting not-*p* for *p* in this formula is, therefore, more than a simple substitution. It involves a species of *syllogism.*

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[3] I must guard against misunderstanding here. So long as the meaning of the symbols is left out of account, this rule, like the other rules for manipulation, is indeed used. It is used as a premise, and deductions are drawn from it. When the symbols are given their logical interpretation, the rule might, indeed, continue to be used as a premise. But it would be obviously superfluous and ineffective; for, using it as a premise, one would have to do the same amount of spontaneous, undirected thinking that its use was supposed to obviate. Accordingly, the authors of "Principia Mathematica" never dream of using it as a premise. And, as we have seen, its use as a formula in which substitution is to be made does not suffice.

[4] In his encyclopedia article, he writes: "What is true of all is true of one in particular. This axiom is the well-known principle of all particular applications of general theorems" (p. 151).

To make this clear I must go into some technicalities, for the full statement of the syllogism that is involved will differ according as one rejects or accepts the "theory of types" which "Principia Mathematica" advocates. If that theory be rejected, the original formula will have to be expanded so as to read: "If *p* is an elementary proposition,[5] then '*p* or *p*' implies *p*." This would then hold, no matter what is put for *p*. Substituting not-*p* for *p,* we have: "If not-*p* is an elementary proposition, 'not-*p* or not-*p*' implies not-*p*." This, however, is not the result required. To obtain it we need the premise, "If *p* is an elementary proposition, then not-*p* is an elementary proposition." This combined with the preceding gives the required formula: "If *p* is an elementary proposition, 'not-*p* or not-*p*' implies not-*p*."

If the theory of types is accepted, the condition, "If *p* is an elementary proposition" need not be written. For, according to this theory, the truth of a general formula does not require that no matter what particular value is assigned to its variables the result must be a true proposition, but only that it shall be either true or nonsensical; and the conventions that are made with regard to the use of the terms "not" and "or" (and consequently with regard to "implies," which is defined in terms of "not" and "or") are such that this formula is nonsense except when *p* is an elementary proposition. The formula is, therefore, asserted unconditionally; for it is true for any interpretation of the symbol *p* for which it has a meaning. Anything whatsoever may, therefore, be substituted for *p* without danger of error. But that the substitution shall have any logical importance it must be known that the new formula is significant under the same condition as the old. By simple substitution, then, we write: " 'Not-*p* or not-*p*' implies not-*p*," and know that this is significant when not-*p* is an elementary proposition. By postulate * 1.7 we know that if *p* is an elementary proposition not-*p* is also. Hence we infer that the new formula also is significant when *p* is an elementary proposition.

On the whole, then, we may say that though the formula in which substitution is made is not a *premise* (as the term is used in "Principia Mathematica," as well as throughout this paper), the complete process of substitution does, in such a case as the above, involve syllogistic reasoning from premises to a conclusion.

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[5] As a matter of fact, if the theory of types be rejected, one will naturally prefer to state the wider formula: "If *p* is a proposition, '*p* or *p*' impies *p*." But I follow "Principia Mathematica" as closely as possible.

I will conclude by quoting with a word of comment a passage from "Principia Mathematica" (Vol. I, p. 94): "The purpose of the present section [Part I, Section A] is to set forth the first stage of the deduction of pure mathematics from its logical foundations. The first stage is necessarily concerned with deduction itself, *i. e.*, with the principles by which conclusions are inferred from premises. If it is our purpose to make all our assumptions explicit, and to effect the deduction of all our other propositions from these assumptions, it is obvious that the first assumptions we need are those that are required to make deduction possible." If deduction were not already possible, no array of assumptions could make it so.

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PRACTICAL VERSUS LITERAL TRUTH

THAT there exist in our current discussions and preachments many ambiguities due to needless inexactness of statement, and others due to the use of identical words in differing senses, we are all painfully aware. At best we may hope that time and a more general education will gradually produce a more accurate speech and a wider agreement in definition. But there is another form of ambiguity which it is not so easy to combat, which we may indeed consent to tolerate, because there seems no way to avoid it without an actual loss to mankind. This ambiguity consists in the danger of taking a practical truth for a literal truth. By a "practical truth" I mean a statement which, although not literally exact, yet enshrines a genuine insight, conveys a useful message. The point I wish to make is that the effectiveness of such assertions would often be seriously impaired by such an alteration of phraseology as would remove the ambiguity; and that, therefore, since clearness of thought is not the only, or indeed the greatest, good, it may be wise to suffer these ambiguities rather than to dim the valuable insights which the ambiguous utterances contain.

Ambiguity does not arise when an assertion is obviously metaphorical, exaggerated, or a short-cut expression. When we say, "Architecture is frozen music"; when Emerson says, "Things are in saddle and ride mankind"; when Jesus says, "He that loseth his life shall find it," we understand the metaphor and run no risk of taking the statement literally. When Emerson writes, "What Plato has thought [every man] may think; what a saint has felt he may feel; what at any time has befallen any man, he can understand," we